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ATMOSPHERIC DENSITY MODELS

ANALYTICAL AND COMPUTATIONAL MATHEMATICS, INC.



ATMOSPHERIC DENSITY MODELS

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ATMOSPHERIC DENSITY MODELS

by

Alan C. Mueller

1.0 Introduction

To compute the density at any point in the atmosphere, the Mission Control Center orbit prediction program will employ a model developed by Jacchia (Reference 1). Although the model is quite accurate, it requires a large amount of computer storage and execution time. Because of software constraints, these undesirable features make the Jacchia model ill-suited for the Shuttle onboard program. However if a different model is to be used on board, mission constraints require a certain degree of compatibility with the Jacchia model used on the ground. Thus the intent of this study is to develop a simple atmospheric density model that simulates the Jacchia model.

2.0 Atmospheric Density Models

To date, there exist only two analytical, dynamic models which include most or all of the important characteristics of the earth's atmospheric density. One is the Jacchia model mentioned in the introduction and the other is a model developed in the U.S.S.R. (Reference 2). The USSR model was used during ASTP, the joint Russian-American mission. The following characteristics of the density are included in both models:

1. Variation with solar activity (daily and 3 month average)
2. Diurnal variation
3. Variation with geomagnetic activity (yearly mean and 3 hourly index)

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4. Semi-annual variation
5. Variation with height

In the Jacchia model, the first four variations are computed as changes in the exospheric temperature T_∞ . One may then obtain a rather complicated solution to the diffuse equilibrium equation, knowing T_∞ and the height h . The density ρ may be determined from the solution of this equation for the different gas constituents. In addition the Jacchia model also computes the variation in density due to a seasonal - latitudinal effect which becomes important for altitudes below 300 Km.

The USSR model, on the other hand, computes all the above characteristics directly as variations in the density and is computationally much simpler that the Jacchia model. The USSR model does not account for the seasonal - latitudinal variations. Unlike the Jacchia model, the coefficients used in the USSR model are not constants but are a function of the changing solar activity level. The coefficients are given in a tabular form for different values of the 10.7 cm solar flux ($F_{10.7}$) , a measure of the solar activity. Unfortunately, the coefficients are not known for the expected values of $F_{10.7}$ in the early 1980's which renders the model useless for many shuttle flights. Yet tests have shown that the USSR model agrees with the Jacchia model for solar activity levels in which the coefficients are known (Section 2.2.2, Reference 3). Therefore the USSR model is certainly a candidate for the on-board program if the coefficients can be determined for the expected values of $F_{10.7}$ during the Shuttle flights.

One approach to determine the coefficients would be by a least square data fit using some iteration scheme such as the Davidon technique to adjust the coefficients. The data to be fit could be obtained from the Jacchia model. Although this type of approach could be extremely accurate it also would require a considerable theoretical and computational effort.

AMDB, a density model developed by Gus Babb (Section 2.2.4, Reference 3) may offer some insight in the problem of determining coefficients for the USSR model. The AMDB model does account for the height and diurnal variations of the density model. The other variations are included implicitly by calibration to the Jacchia model. As in the USSR model, the coefficients in AMDB, are implicit functions of the solar activity level, but unlike the USSR model, AMDB coefficients may be determined easily by a procedure known as calibration. The AMDB model's simple formulation allows one to invert the the problem; given the densities at different points in the atmosphere one can explicitly solve for the coefficients. However, the AMDB model does not explicitly account for all the variations and has been found to be not as accurate as the USSR model (Section 2.2.5, Reference 3)

Thus a new approach is proposed which is to develop a new density model using the accuracy advantages of the USSR model and the calibration advantages of AMDB. With this new direction, it may be possible to reach the goals established in the introduction.

3.0 Comparison of AMDB and USSR models

Before attempting to develop a hybrid of the USSR and AMDB models it is best to understand the similarities and differences between the two models. Since the ground Jacchia model will neglect the short period variations in the solar activity level and geomagnetic index (because of their small effect and for operational reasons), these variations will not be included in the comparison. With these effects neglected, the USSR model reduces to the following form for the density ρ ,

$$\rho = \rho_n \cdot K_1 \cdot K_2 \quad , \tag{1}$$

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where ρ_n is the night time vertical profile given by

$$\rho_n = \exp\left(a_1' - a_2' \sqrt{(h - a_3')}\right) \quad , \tag{2}$$

K_1 is the variation due to the diurnal effect given by

$$K_1 = 1 + B(h) \cdot b\,(\alpha_v, \delta_v, \alpha_s, \delta_s) \tag{3}$$

and K_2 is the variation due to the semi-annual effect given by

$$K_2 = 1 + D(h) \cdot E(d) \tag{4}$$

The function B is a rather complicated function of the altitude above the oblate earth h . D is a linear function of the height and b is a trignometric function of the right ascension and declination of the vehicle (α_v, δ_v) and that of the sun (α_s, δ_s) . And finally E(d) is a tabular function of the day of the year.

It is the diurnal term (equation (3)) which prevents the USSR model from being analytically inverted (given the density solve for the coefficients in the model). Of the 15 coefficients used in this model, 10 are contained in the complicated expression for the diurnal term.

In the AMDB model the density is given by

$$\rho = \rho_o \exp(F + G) \tag{5}$$

where F is the dusk density vertical profile given by

$$F = a_1 + a_2 h + a_3/h \tag{6}$$

and G is the diurnal term

$$G = (b_1 + b_2 h + b_3/h) \cdot g\,(\alpha_v, \delta_v, \alpha_s, \delta_s) \tag{7}$$

where a_1, a_2, a_3, b_1, b_2, b_3 are model parameters.

The major difference between the models is the fact that the diurnal term in AMDB is an additive term within the exponent. This results in an adequate but simple form for the magnitude of the diurnal term as a function of the height. The trignometric function g, however, does not seem to accurately model the angular dependence of the diurnal term.

4.0 The Diurnal Term

From the comparison of the two models it is apparent that the diurnal term results in complications in both models. In the AMDB model it is inaccurate and USSR model it is too complicated.

Assume that the density at a specific height but at arbitrary angular coordinates is given by

$$\rho = \rho_n (1+Bg^*) \qquad \begin{matrix} 0 \leq g^* \leq 1 \\ B \geq 0 \end{matrix} \tag{8}$$

such that when $g^* = 0$, the density is at a minimum and $g^* = 1$ the density is at a maximum. B then gives the difference of the densities at two points at the same height: one directly in the atmospheric bulge where the density is greatest and the other point on the opposite end of the line which passes through the bulge and the center of the earth where the density is least. Assume also that the function g^* varies by the following

$$g^* = \left(\frac{L}{D} \right)^m \tag{9}$$

where L is the distance between the vehicle and the least dense point, D is the distance between the least and most dense points and m will be defined later. Since these points are restricted to a fixed altitude, D is equal to two times the satellite's distance from the center of the earth, $D = 2R$.

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The distance L is given by

$$L = \sqrt{(x-x_m)^2 + (y-y_m)^2 + (z-z_m)^2} \qquad (10)$$

where

x, y, z define the vehicle position

x_m, y_m, z_m define the least dense point

$$x_m = R \cos(\gamma+\pi) \cos(-\delta_s) = -R \cos\gamma \cos\delta_s$$

$$y_m = R \sin(\gamma+\pi) \cos(-\delta_s) = -R \sin\gamma \cos\delta_s \qquad (11)$$

$$z_m = R \sin(-\delta_s) = -R \sin\delta_s$$

$$\gamma = \alpha_s + \phi \qquad (12)$$

ϕ : is the lag of the bulge behind the sun because of rotation of the earth.

Inserting all this into equation (10) and simplifying, one finds

$$L = 2R \left(\frac{1 + \cos\psi}{2} \right)^{\frac{1}{2}}$$

where

$$\cos\psi = \frac{1}{R} \left[z \sin\delta_s + \cos\delta_s (x \cos\gamma + y \sin\gamma) \right] \qquad (13)$$

and thus

$$g^* = \left(\frac{L}{D}^m \right) = \left(\frac{1+\cos\psi}{2} \right)^{\frac{m}{2}} \qquad (14)$$

Finally by trignometric identities one obtains

$$g^* = \left[\cos\frac{\psi}{2} \right]^m \qquad (15)$$

By careful observation of the diurnal function in the USSR one finds it has the form

$$b = \cos^{m_1} \frac{\psi_1}{2} + c \cos^{m_2} \frac{\psi_2}{2} \tag{16}$$

where ψ_1 is the angle formed by the least dense point and the vehicle point and the center of the earth. ψ_2 is the angle for the most dense point. But c is always small ($|c| \leq .05$) and if this term is neglected the angular function in the diurnal term is exactly that obtained in equation (15). With all this in mind we now attempt to define a hybrid AMDB/USSR model.

5.0 Development of Hybrid Model

From considerations of sections 3 and 4 the following model is proposed

$$\rho^* = \rho_o \exp (A + B) \tag{17}$$

where A is the night time vertical profile

$$A = a_1 + a_2 h + \frac{a_3}{h} \tag{18}$$

and B is the diurnal effect

$$B = (b_1 + b_2 h + \frac{b_3}{h}) (\frac{1+\cos\psi}{2})^{\frac{m}{2}} \tag{19}$$

where $\cos \psi$ is given by equation (13).

The seasonal-latitudinal variation may be included in this new model. Since the Jacchia model computes this term as an explicit variation in the density one may incorporate this in the new model easily. The seasonal-latitudinal variation that is given by Jacchia is

$$\rho = \rho^* \cdot 10^{\varepsilon} \tag{20}$$

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where

$$\varepsilon = (0.02)\ (h-90)\ \frac{\delta_v}{|\delta_v|}\ \exp\left[\ -0.045\ (h-90)\ \right]\ \sin^2 \delta_v \cdot \sin\left[\ \frac{360}{Y}\ (d+100)\ \right] \tag{21}$$

h is given in km, d is the number of days into the year and Y is the number of days in a year. This term may be rewritten so that the final form of the model is

$$\rho = \rho_o \exp\ (A+B+C) \tag{22}$$

where

$$C = .04605 * |Z| \cdot \frac{Z}{R^2}\ (h-90)\ \exp\left[\ -0.045\ (h-90)\ \right] \cdot \sin\left[\ \frac{360}{Y}\ (d+100)\ \right] \tag{23}$$

The term underlined may be computed once with the initial day d and days in the year Y and then assumed constant.

6.0 Calibration Procedure

If one assumes that the coefficient m in equation (14), ρ_o and the bulge angle ϕ are known constants and unaffected by the solar activity or geomagnetic changes then the hybrid model has 6 coefficients which are determined through calibration to the Jacchia model.

Let us first give the general solution to 3 equations in 3 unknowns with the following form

$$c_1 + c_2 w_i + c_3/w_i = f_i \qquad i = 1, 2, 3 , \quad (24)$$

The solution of these equations is

$$c_3 = \left[f_3-f_1 + \frac{w_1-w_3}{w_2-w_1}(f_2-f_1)\right] \left[(w_1-w_3)\left(\frac{1}{w_1w_3} - \frac{1}{w_1w_2}\right)\right] ,$$

$$c_2 = \frac{f_2-f_1}{w_2-w_1} + \frac{c_3}{w_1w_2} , \quad (25)$$

$$c_1 = f_1 - c_2w_1 - c_3/w_1 .$$

If one choses the night time minimum density point in space defined by

$$\alpha_v = \gamma + \pi$$
$$\delta_v = - \delta_s$$

then the diurnal term B is equal to zero. Thus if three altitudes are chosen, h_1, h_2, and h_3, then the coordinates for the three altitudes where the diurnal term is zero are given by

$$x_i = (R_e+h_i) \cos(\gamma+\pi) \cos(-\delta_s) ,$$
$$y_i = (R_e+h_i) \sin(\gamma+\pi) \cos(-\delta_s) , \quad (26)$$
$$z_i = (R_e+h_i) \sin(-\delta_s) ,$$

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where R_e is the radius of the earth. Given the coordinates for the 3 different altitudes, the solar flux intensity, the geomagnetic index and the day of the year, one may determine three densities from the Jacchia model. One then defines the function f_i as

$$f_i = (\ln \rho_i/\rho_c - C_i) \quad \text{for} \quad i = 1, 2, 3 \qquad (27)$$

where ρ_i is the density determined from Jacchia evaluated with the ith position and C_i is found from evaluating equation (23) with the ith position. If one then defines

$$w_i = h_i \qquad \text{for } i = 1, 2, 3 \qquad (28)$$

then coefficients of the night time vertical profile may be determined by equation (25) where

$$a_k = c_k \qquad \text{for } k = 1, 2, 3 \quad . \qquad (29)$$

The coefficients in the diurnal term may now be determined by choosing the coordinates of the vehicle such that $(\frac{1+cos\ \psi}{2})^{\frac{m}{2}}$ is equal to 1 (daytime maximum density point). This is the case when

$$\begin{aligned} x_i &= (R_e+h_i)\ cos\ \gamma\ cos\ \delta_s \\ y_i &= (R_e+h_i)\ sin\ \gamma\ cos\ \delta_s \\ z_i &= (R_e+h_i)\ sin\ \delta_s \end{aligned} \qquad (30)$$

One then may define a new f_i

$$f_i = \ln (\rho_i/\rho_o) - (A_i + C_i) \qquad \text{for } i = 1, 2, 3 \quad (31)$$

where ρ_i is the density determined from Jacchia evaluated with the ith position, and C_i and A_i are determined from evaluating equation (23) and equation (18) respectively with the ith position. If one then defines

$$w_i = h_i \qquad \text{for } i = 1, 2, 3 \tag{32}$$

the diurnal coefficients may be determined from equation (25) where

$$b_k = c_k \qquad \text{for } k = 1, 2, 3 \quad . \tag{33}$$

The proper altitudes to choose may be defined by the region in which one wishes to use the model. In the numerical experiments in Section 7, the following heights were chosen: $h_1 = 150$, $h_2 = 300$, $h_3 = 450$ km. If the model is to be used over a large range of heights then one may layer the night time profile in two sections.

By careful study of the Jacchia model a value of $\phi = 37^0$ (equation (12)) and $m = 2.75$ (equation (19)) has been adopted. ρ_o (equation (22)) has the value of 1.224997 kg/m^2. However all these values may be refined to obtain closer agreement to the Jacchia model.

7.0 Numerical Experiments

To compare the new density model to the Jacchia* model, several numerical experiments have been carried out. Instead of comparing directly the computed densities, predicted satellite positions are compared from a numerical orbit computation program (Reference 4). The density model is used to compute the drag forces on the satellite. Several different orbits were chosen for the comparison. Thus the position difference is a good indication of the global difference between

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the density models. The orbits chosen are the same as in Section 2.2.5, Reference 3. Also the ballistic number is an average value for the shuttle, BN = 100 $lb/(ft)^2$, and the coefficients of drag, C_d , is set to 2.2. Table I displays the chosen orbits and Table II gives the results. Also included are the results of the USSR and AMDB model. The hybrid model will henceforth be called the Babb-Mueller (B-M) model. Gus Babb, of FPD, originated the AMDB model and this author has included the additional terms from the USSR model. The results in Table II are the differences in predicted position using the Jacchia model as compared to using the other three models or neglecting drag completely.

The coefficients used in the USSR model correspond to an $\overline{F}_{10.7} = 75$. This happens to be the actual $\overline{F}_{10.7}$ which occured in 1975. Thus one would expect that the USSR and Jacchia models to agree well in the year 1975. For all these cases in Table II the USSR model does show close agreement with Jacchia, but the new model (B-M) shows better agreement with Jacchia than any of the other models, including the USSR model. Case E has an epoch of 1977 which corresponds to a $\overline{F}_{10.7} = 110$. The USSR model still used the coefficients of $\overline{F}_{10.7} = 75$. Thus one expects to see a disagreement to Jacchia. In Table II, one sees that this is the case. However, the B-M model still shows a close agreement.

* In the Jacchia model, the daily changes in F_{10} have been neglected and also the hourly changes in the geomagnetic index (a_p) are dropped.

TABLE I: ORBITS USED IN PREDICTION EXPERIMENTS

	A	B	C	D	E
perigee (km)	220	300	300	220	Same as
apogee (km)	380	600	600	380	A except
eccentricity	.012	.022	.022	.012	epoch
period (min)	90.5	93.6	93.6	90.5	is
argument of perigee	0	0	180^0	0	12:00
ascending node	0	0	0	0	January 1,
inclination	30^0	30^0	30^0	90^0	1977
epoch	12:00	January 1, 1975			

TABLE II: POSITION DEPENDENCE ON DENSITY MODEL

Time of integration (days)	Position difference (km)			
	No Drag	USSR	AMDB	B-M
0.5	8.9	0.2	0.3	.005
1.0	36.1	0.8	1.5	.018

(a) Orbit A

TABLE II: CONTINUED

Time of integration (days)	Position difference (km)			
	No Drag	USSR	AMDB	B-M
0.5	.83	.03	.07	.004
1.0	3.16	.07	.28	.015

(b) Orbit B

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TABLE II: CONTINUED

Time of integration (days)	Position difference (km)			
	No Drag	USSR	AMDB	B-M
0.5	.5	<.001	.01	.004
1.0	1.8	.02	.04	.018

(c) Orbit C

TABLE II: CONTINUED

Time of integration (days)	Position difference (km)			
	No Drag	USSR	AMDB	B-M
0.5	8.6	.1	.3	.008
1.0	34.4	.5	1.5	.027

(d) Orbit D

TABLE II: CONTINUED

Time of integration (days)	Position difference (km)			
	No Drag	USSR	AMDB	B-M
0.5	13.9	3.4	0.4	.071
1.0	56.0	19.2	1.9	.275

(e) Orbit E

8.0 Conclusions and Outlook

From the results of the numerical experiments it is concluded that the B-M model gives better agreement to the Jacchia than either the USSR or AMDB model. This demonstrates the validity of the chosen approach. Additional studies may be conducted to determine the range of altitudes over which the model is valid and whether a layering of the atmosphere model is necessary. Also, additional studies need to be made to refine the angular parameter (ϕ) which defines the position of the atmospheric bulge and the power exponent (m) in the diurnal term.

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